Introduction to Multidimensional Scaling

Measurement, Scaling, and Dimensional Analysis
2019 ICPSR Summer Program
Prof. Adam M. Enders

Announcements

- Schedule:
  - Tuesday–Wednesday: metric and nonmetric (ordinal) MDS
  - Thursday: MDS extensions (unfolding, WMDS, vector model)
  - Friday: (multiple) correspondence analysis
- Blalock lecture tonight @ 7 PM in Angell Hall, Auditorium C
  - "Studying Racial and Gender Inequality: It Matters How You Measure," Aliya Saperstein (Stanford University)
- Factor analysis homework due **tomorrow** (Wednesday)

An Example

- Assume we have information about the American electorate's perceptions of thirteen prominent political figures from the period of the 2004 presidential election
- Specifically, we have the perceived **dissimilarities** between all pairs of political figures
  - With 13 figures, there will be 78 distinct pairs of figures
  - Rank-order pairs of political figures, according to their dissimilarity (from least to most dissimilar)
- For convenience, arrange the rank-ordered dissimilarity values into a square, symmetric matrix

Matrix of Perceptual Dissimilarities

gwbush	0.0	73.0	62	8.0	68.0	20.0	51.5	41.0	24.0	7	25.5	50	5.0
kerry	73.0	0.0	56	78.0	1.0	54.0	15.0	17.0	47.0	77	37.0	2	74.5
nader	62.0	56.0	0	72.0	59.0	53.0	60.0	49.0	58.0	70	39.0	57	71.0
cheney	8.0	78.0	72	0.0	74.5	25.5	65.0	51.5	29.0	12	30.0	66	4.0
edwards	68.0	1.0	59	74.5	0.0	44.0	14.0	16.0	46.0	76	38.0	3	69.0
lbush	20.0	54.0	53	25.5	44.0	0.0	42.0	34.0	9.5	23	22.0	45	18.0
hclinton	51.5	15.0	60	65.0	14.0	42.0	0.0	19.0	32.0	67	40.0	13	55.0
bclinton	41.0	17.0	49	51.5	16.0	34.0	19.0	0.0	31.0	61	36.0	11	48.0
powell	24.0	47.0	58	29.0	46.0	9.5	32.0	31.0	0.0	28	9.5	35	21.0
ashcroft	7.0	77.0	70	12.0	76.0	23.0	67.0	61.0	28.0	0	33.0	63	6.0
mccain	25.5	37.0	39	30.0	38.0	22.0	40.0	36.0	9.5	33	0.0	43	27.0
dempty	50.0	2.0	57	66.0	3.0	45.0	13.0	11.0	35.0	63	43.0	0	64.0
reppty	5.0	74.5	71	4.0	69.0	18.0	55.0	48.0	21.0	6	27.0	64	0.0

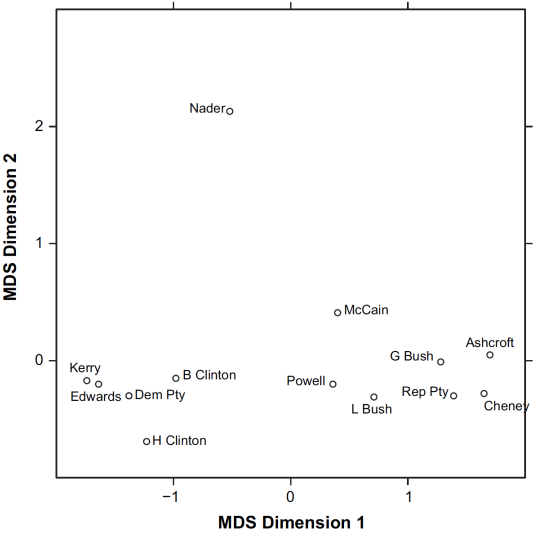
Clearly, there is too much information in this matrix to be comprehensible in its “raw” numeric form

Instead, try “drawing a picture” of the information in the matrix

Rules for Drawing a Picture

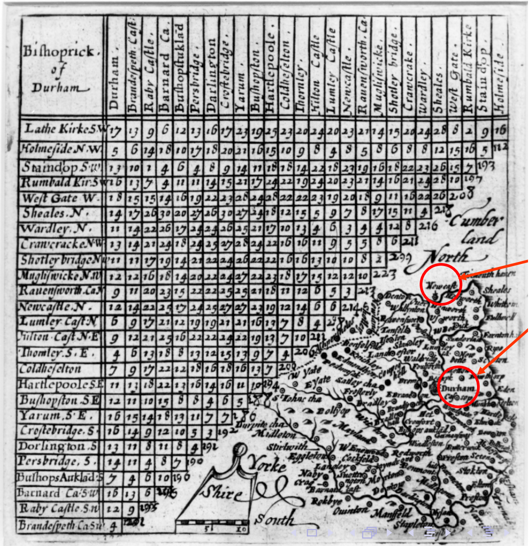
- Each political figure is shown as a point
- Points are located on the surface of the display medium
  - For example, the projection screen or a sheet of paper (a two-dimensional space)
- Adjust the point locations so that the rank-order of the distances between pairs of points corresponds as closely as possible to the rank-ordered dissimilarities between pairs of political figures
- The process of constructing the picture from the dissimilarities is **multidimensional scaling**

MDS Configuration of Political Figures



The First MDS?

Cartographer, Jacod van Langren, produces a distance matrix and a map on Durham County, England (1635)



A General Definition of MDS

- A family of procedures for constructing a spatial model of objects, using information about the proximities between the objects
- Different "varieties" of MDS
  - Classical multidimensional scaling (CMDS)
  - Weighted multidimensional scaling (WMDS)
  - Unfolding models (or "ideal point models")
  - Preference mapping (or "the vector unfolding model")
- Even basic correspondence analysis (CA) can be thought of as a special kind of multidimensional scaling (of a cross-tabulation table)

Utility of MDS for Social Research

- Reducing dimensionality
  - Essentially built to produce two-dimensional maps of spatial relationships
- Flexible with respect to input data
  - Classical MDS is interested in **square**, **proximity** data
  - Unfolding models are useful for **rectangular**, **proximity** data
  - The way we get to proximity data, or even shape, can be "creative" (just like any of the methodologies we've discussed so far)
- Useful measurement tool
  - Like other models, MDS models help us move "up" a level of measurement and estimate the dimensions we're really interested in
- Graphical output
  - Statistically graphics are worth a million numbers!
  - Anyone can understand output based on interpoint distances

The Map Analogy

- A familiar task:
  - Starting with a map (a geometric model)
  - Can obtain the distances between locations (numeric data)
- MDS "reverses" the preceding task:
  - We begin with distances (numeric data)
  - Use that information to produce a map (geometric model)

More Formally...

- Begin with a $k \times k$ symmetric matrix, Δ, of "proximities" among k objects
  - The proximity between the object represented by the i^{th} row and the object represented by the j^{th} column is shown by the cell entry, δ_{ij}
  - Greater proximity between objects i and j corresponds to smaller values of δ_{ij} and vice versa
  - Therefore, the proximities are often called "dissimilarities"
  - Admittedly, this terminology is a bit confusing!
  - But, there is a reason for it...

More Formally..., cont'd

- MDS tries to find a set of k points in m-dimensional space such that the distances between pairs of points approximate the dissimilarities between pairs of objects
- More specifically, MDS uses the information in $\mathbf{\Delta}$ to find a $k \times m$ matrix of coordinate values, $\mathbf{X}$
- The distance between the points representing objects i and j, d_{ij}, is calculated from the entries in the i^{th} and j^{th} rows of $\mathbf{X}$, using the Pythagorean formula:

$$d_{ij} = \left[(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2 \ldots + (x_{im} - x_{jm})^2\right]^{0.5}$$

- We want MDS to find $\mathbf{X}$ such that $d_{ij} \approx \delta_{ij}$ for all i and j

More Formally..., cont'd

- Stated a bit differently:
  - MDS uses the information contained in $\mathbf{\Delta}$ to find $\mathbf{X}$ such that the interpoint distances are functionally related to the pairwise dissimilarities
  - For all pairs, i and j, with $i \neq j$:

$$d_{ij} = f(\delta_{ij})$$

  - The nature of the function, f , is determined by the type of MDS that is performed on the dissimilarities data

Metric MDS

- Metric multidimensional scaling requires that distances are related to dissimilarities by a linear function:

$$d_{ij} = a + b\delta_{ij} + e_{ij}$$

- In the preceding formula, a and b are coefficients to be estimated, and e_{ij} is an error term associated with objects i and j
- Stated differently, metric MDS assumes that the input dissimilarities data are measured at the interval or ratio level (if we remove the constant, a, by setting it to 0)
- We will also discuss ordinal or "nonmetric," as it is most frequently referred, MDS
  - In this case, the distances are monotonically related to the dissimilarities
  - Something like: $d_{ij} = f^m(\delta_{ij}) + e_{ij}$

Torgerson's Procedure for Metric MDS

- Begin with $k \times k$ dissimilarities matrix, $\boldsymbol{\Delta}$
- Assume the dissimilarities correspond to distances in m-dimensional space (except for random error)
  - The $k \times k$ matrix of distances between points is $\mathbf{D}$
  - The $k \times k$ matrix, $\mathbf{E}$, contains random errors
- Hypothesis:
  - $\mathbf{D} = \boldsymbol{\Delta} + \mathbf{E}$
- Objective:
  - Find the $k \times m$ coordinate matrix, $\mathbf{X}$, such that entries in $\mathbf{E}$ are as close to zero as possible

Torgerson's Procedure for Metric MDS, cont'd

- Create a "double-centered" version of $\mathbf{\Delta}$, designated $\mathbf{\Delta}^*$
  - Double-centering transforms $\mathbf{\Delta}$, so that the row sums, the column sums, and the overall sum of the cell entries in the matrix are all zero
  - For dissimilarity δ_{ij} , the corresponding entry in the double-centered matrix is calculated as follows:

$$\delta^*_{ij} = -0.5(\delta^2_{ij} - \delta^2_{i.} - \delta^2_{.j} + \delta^2_{..})$$

- $\mathbf{\Delta}^*$ can be factored to obtain the matrix of point coordinates:

$$\mathbf{\Delta}^* = \mathbf{X}\mathbf{X}'$$

- The factoring process is somewhat analogous to obtaining a square root

Torgerson's Procedure for Metric MDS, cont'd

- The factoring process is carried out by performing an eigendecomposition on $\mathbf{\Delta}^*$

$$\mathbf{\Delta}^* = \mathbf{V}\mathbf{\Lambda}^2\mathbf{V}'$$

- $\mathbf{V}$ is the $k \times q$ matrix of eigenvectors, $\mathbf{\Lambda}^2$ is the $q \times q$ diagonal matrix of eigenvalues, and q is the rank of $\mathbf{\Delta}^*$ (usually equal to k)
- Create $\mathbf{X}$ from the first m eigenvectors ($\mathbf{V}_m$) and the first m eigenvalues ($\mathbf{\Lambda}^2_m$):

$$\mathbf{X} = \mathbf{V}_m\mathbf{\Lambda}_m$$

- $\mathbf{X}$ contains point coordinates such that the interpoint distances have a least-squares fit to the entries in $\mathbf{\Delta}$

Example: Distances Between Cities

Driving distances between 10 cities (in thousands of miles):

0	0.587	1.212	0.701	1.936	0.604	0.748	2.139	2.182	0.543	ATLANTA
0.587	0	0.920	0.940	1.745	1.188	0.713	1.858	1.737	0.597	CHICAGO
1.212	0.920	0	0.879	0.831	1.726	1.631	0.949	1.021	1.494	DENVER
0.701	0.940	0.879	0	1.374	0.968	1.420	1.645	1.891	1.220	HOUSTON
1.936	1.745	0.831	1.374	0	2.339	2.451	0.347	0.959	2.300	LOS ANGELES
0.604	1.188	1.726	0.968	2.339	0	1.092	2.594	2.734	0.923	MIAMI
0.748	0.713	1.631	1.420	2.451	1.092	0	2.571	2.408	0.205	NEW YORK
2.139	1.858	0.949	1.645	0.347	2.594	2.571	0	0.678	2.442	SAN FRANCISCO
2.182	1.737	1.021	1.891	0.959	2.734	2.408	0.678	0	2.329	SEATTLE
0.543	0.597	1.494	1.229	2.300	0.923	0.205	2.442	2.329	0	WASHINGTON DC

The preceding $\boldsymbol{\Delta}$ is a good example for metric MDS:

- We already know m (the dimensionality)
- We already know the "shape" of the point configuration

Example: Distances Between Cities

The following matrix is the $\mathbf{\Delta}^*$ obtained when Torgerson's transformation is applied to all cells of the intercity distance matrix, $\mathbf{\Delta}$

0.537	0.228	-0.348	0.199	-0.808	0.895	0.697	-1.005	-1.050	0.656
0.228	0.263	-0.174	-0.134	-0.594	0.234	0.585	-0.581	-0.315	0.488
-0.348	-0.174	0.236	-0.092	0.570	-0.563	-0.504	0.681	0.658	-0.463
0.199	-0.134	-0.092	0.352	0.029	0.516	-0.124	-0.163	-0.550	-0.033
-0.808	-0.594	0.570	0.029	1.594	-1.130	-1.499	1.751	1.399	-1.313
0.895	0.234	-0.563	0.516	-1.130	1.617	0.920	-1.542	-1.867	0.918
0.697	0.585	-0.504	-0.124	-1.499	0.920	1.416	-1.583	-1.130	1.222
-1.005	-0.581	0.681	-0.163	1.751	-1.542	-1.583	2.028	1.846	-1.432
-1.050	-0.315	0.658	-0.550	1.399	-1.867	-1.130	1.846	2.124	-1.115
0.656	0.488	-0.463	-0.033	-1.313	0.918	1.222	-1.432	-1.115	1.071

An eigendecomposition is carried out on the preceding $\mathbf{\Delta}^*$

We expect a two-dimensional solution, so we use the first two eigenvectors and the first two eigenvalues

Example: Distances Between Cities

First two eigenvectors of double-centered data matrix, $\mathbf{\Delta}^*$:

-0.23217	-0.11011
-0.12340	0.26253
0.15554	0.01929
-0.05216	-0.44079
0.38889	-0.30037
-0.36618	-0.44802
-0.34640	0.39964
0.45892	-0.08658
0.43346	0.44649
-0.31645	0.25843

First two eigenvalues of double-centered data matrix, $\mathbf{\Delta}^*$:

9.58217	1.68664

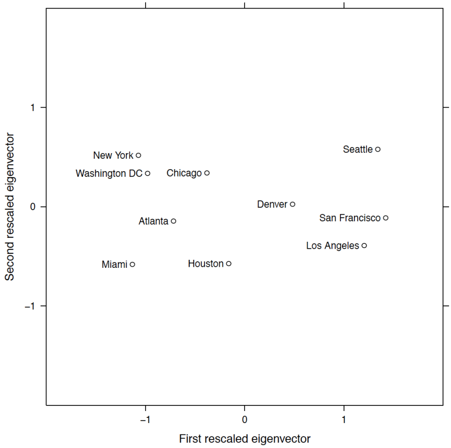
Example: Distances Between Cities

The eigenvectors are multiplied by the square roots of the corresponding eigenvalues to produce the matrix of two-dimensional point coordinates: $\mathbf{X} = \mathbf{V}_2 \mathbf{\Lambda}_2$

-0.71867	-0.14300	Atlanta
-0.38197	0.34095	Chicago
0.48149	0.02505	Denver
-0.16147	-0.57246	Houston
1.20382	-0.39009	Los Angeles
-1.13352	-0.58185	Miami
-1.07228	0.51901	New York
1.42058	-0.11244	San Francisco
1.34179	0.57986	Seattle
-0.97958	0.33562	Washington D.C.

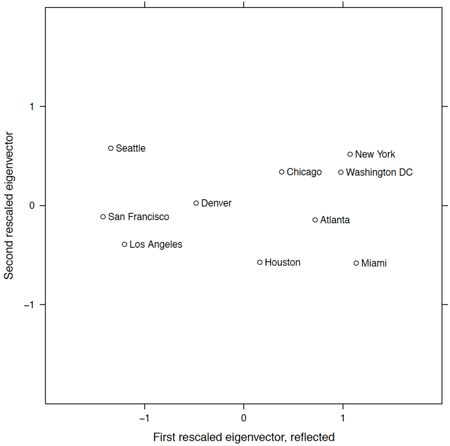
Point coordinates can be plotted in two-dimensional space

Example: Distances Between Cities



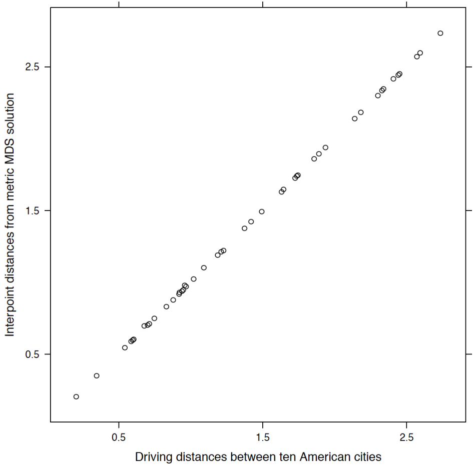
Interpoint configuration looks good, but western cities are on the right

Example: Distances Between Cities



Can "reflect" the first dimension by multiplying coordinates by -1

Example: Distances Between Cities

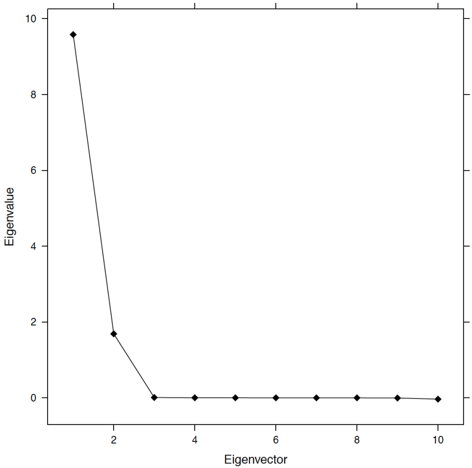


Can graph scaled interpoint distances versus input dissimilarities data (i.e., driving distances)

Model Fit: Graphical Assessment

- Eigenvalues are related to the variance in the double-centered distances that is "explained" by each eigenvector
- The "Scree plot" shows the eigenvalues plotted in the order that they were factored from the dissimilarities matrix
- Useful visual representation of model fit
- Logic:
  - The "important" dimensions in a metric MDS solution should account for a large part of the variance in the dissimilarities data
  - Dimensions associated primarily with error should account for very little variance

Scree Plot for Metric MDS Solution



Graph of eigenvalues versus order of extraction in metric MDS of intercity driving distances

Goodness of Fit Measure for Metric MDS

- Eigenvalues measure variance associated with each dimension of the MDS solution
- Sum of first m eigenvalues relative to sum of all q eigenvalues (usually $q = k$):

$$Fit = \frac{\sum_{i=1}^{m} \lambda_i^2}{\sum_{i=1}^{q} \lambda_i^2}$$

- *Fit* is bound between (0, 1) with 1 denoting "perfect" fit
- Here, first two eigenvalues are 9.58 and 1.69, and the sum of the eigenvalues is 11.32

$$Fit = \frac{9.59 + 1.69}{11.32} = 0.996$$

A Conceptual Leap

- So far, we have used physical distances as the input data
- If MDS works for physical distances, then it may also work for data that can be interpreted as “conceptual distances”
- Many types of data can be interpreted as conceptual distances
  - Usually correspond to ideas like “proximity” and similarity (or dissimilarity)
  - We will say more about this later...

Profile Dissimilarities

- One important type of conceptual distance data:
  - Each of k objects has scores on each of v variables
  - Each object's vector of scores is called its *profile*
  - For each pair of objects, take the sum of squared differences across the v variables and, optionally, take the square root of the sum
  - For objects i and j, each of which have scores on variables $x_1, x_2, \ldots x_v$, the profile dissimilarity is:

$$\delta_{ij} = \Big[\sum_{l=1}^{v} (x_{il} - x_{jl})^2 \Big]^{0.5}$$

  - δ_{ij} is the profile dissimilarity between the two objects; it can be interpreted as the distance between them in v-dimensional space.

Example: Same Ten Cities

Socioeconomic characteristics of 10 American cities:

	Climate, Terrain	Housing	Environ., Health	Crime	Transport- ation	Education	The Arts	Recreation	Economics
Atlanta	0.185	-1.338	-0.451	-0.609	0.817	-0.413	-0.700	-1.352	0.327
Chicago	-0.942	-0.350	0.977	-1.139	0.423	1.112	0.431	-0.201	-1.142
Denver	-0.899	-0.397	-0.820	-0.498	0.661	-0.100	-0.640	-0.611	1.451
Houston	-1.500	-0.789	-0.856	-0.239	-1.419	0.347	-0.470	-0.995	1.848
LA	1.356	0.774	0.636	0.652	-1.585	0.077	0.347	0.640	-0.995
Miami	-0.198	-0.596	-0.941	1.617	-1.078	-1.046	-0.879	0.911	-0.301
NYC	-0.174	0.580	2.135	1.692	0.945	-0.673	2.520	0.355	-0.966
SF	1.511	2.026	-0.156	-0.007	0.752	0.703	-0.264	1.142	-0.006
Seattle	0.879	-0.628	-0.718	-0.876	-0.231	-1.647	-0.568	1.297	-0.220
DC	-0.217	0.719	0.196	-0.591	0.715	1.642	0.225	-1.186	0.005

Table entries are standardized versions of scores assigned to cities in *Places Rated Almanac*, by Richard Boyer and David Savageau. For all but two of the above criteria, the higher the score, the better. For Housing and Crime, the lower the score the better.

Use this information to calculate the 10×10 matrix, $\boldsymbol{\Delta}$, of profile dissimilarities between the cities

Example: Same Ten Cities

Profile dissimilarities matrix, $\boldsymbol{\Delta}$:

Atlanta	0.000	3.438	2.036	3.394	4.630	3.930	5.555	4.615	3.330	3.168
Chicago	3.438	0.000	3.635	4.364	3.964	4.740	4.357	4.253	4.299	2.296
Denver	2.036	3.635	0.000	2.333	4.849	3.794	5.674	4.285	3.606	2.994
Houston	3.394	4.364	2.333	0.000	5.016	3.959	6.453	5.517	4.588	3.911
Los Angeles	4.630	3.964	4.849	5.016	0.000	3.361	4.181	3.180	3.611	4.039
Miami	3.930	4.740	3.794	3.959	3.361	0.000	5.234	4.471	2.959	4.906
New York	5.555	4.357	5.674	6.453	4.181	5.234	0.000	4.930	5.535	4.796
San Francisco	4.615	4.253	4.285	5.517	3.180	4.471	4.930	0.000	3.894	3.422
Seattle	3.330	4.299	3.606	4.588	3.611	2.959	5.535	3.894	0.000	4.744
Washington DC	3.168	2.296	2.994	3.911	4.039	4.906	4.796	3.422	4.744	0.000

Apply Torgerson's formula to double-center this matrix

Example: Same Ten Cities

The $\mathbf{\Delta}^*$ matrix:

Atlanta	5.668	0.028	3.373	2.289	-4.232	-0.963	-4.318	-3.394	0.925	0.623
Chicago	0.028	6.211	-0.889	-1.201	-1.097	-4.205	1.890	-1.515	-2.498	3.277
Denver	3.373	-0.889	5.226	5.104	-5.490	-0.658	-5.212	-2.143	-0.251	0.940
Houston	2.289	-1.201	5.104	10.430	-3.712	1.303	-7.334	-5.579	-1.674	0.374
Los Angeles	-4.232	-1.097	-5.490	-3.712	7.310	1.931	3.191	3.021	0.771	-1.693
Miami	-0.963	-4.205	-0.658	1.303	1.931	7.851	-1.499	-1.644	3.183	-5.298
New York	-4.318	1.890	-5.212	-7.334	3.191	-1.499	16.554	0.548	-3.404	-0.416
San Francisco	-3.394	-1.515	-2.143	-5.579	3.021	-1.644	0.548	8.849	0.477	1.379
Seattle	0.925	-2.498	-0.251	-1.674	0.771	3.183	-3.404	0.477	7.275	-4.805
Washington DC	0.623	3.277	0.940	0.374	-1.693	-5.298	-0.416	1.379	-4.805	5.619

Use eigendecomposition to obtain metric MDS solution

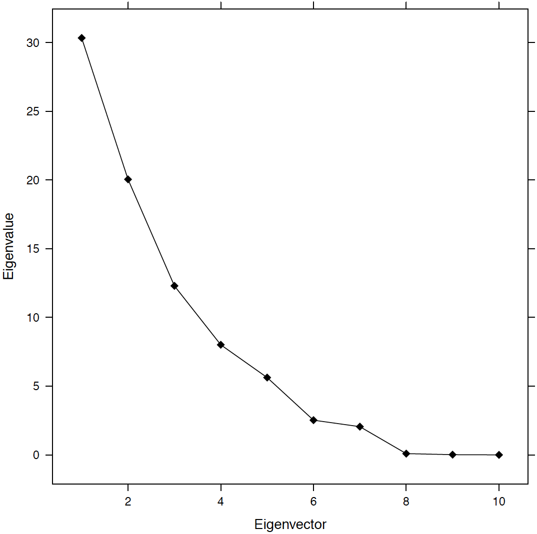
But, an important preliminary question...

Assessing Dimensionality

- What is appropriate dimensionality for the metric MDS solution?
  - Could be any number up to $k - 1$
- Competing considerations:
  - Enough dimensions to account for sufficient variance
  - Relatively few dimensions to facilitate interpretation
- Try examining the scree plot

Evidence from a Scree Plot

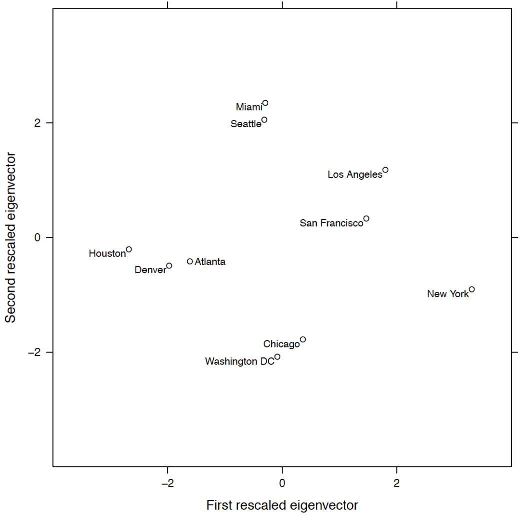
Scree plot for eigendecomposition of $\mathbf{\Delta}^*$ matrix:



Interpreting Scree Plot Evidence

- No obvious "elbow" in scree plot
  - No clear distinction between "important" and "unimportant" dimensions, in terms of variance explained
- Use parsimony for guidance
  - Use $m = 2$ so we can visualize the MDS solution easily

Metric MDS Solution



Goodness of Fit

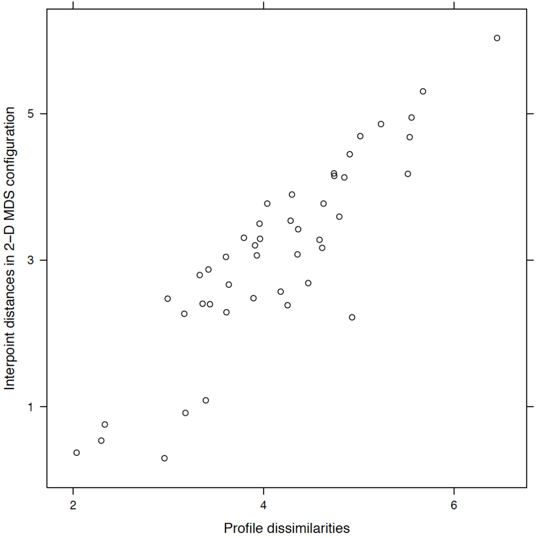
- Calculate fit statistic for the metric MDS solution:

$$Fit = \frac{\sum_{i=1}^{m} \lambda_i^2}{\sum_{i=1}^{q} \lambda_i^2} = \frac{30.337 + 20.046}{81} = 0.622$$

- The two-dimensional MDS solution "explains" about 62% of the variance in the profile dissimilarities
  - Remaining variance is error
  - We have assumed the error is random
  - Is it?

Assessing Errors in Metric MDS Solution

Shepard Diagram for Metric MDS of City Characteristics:



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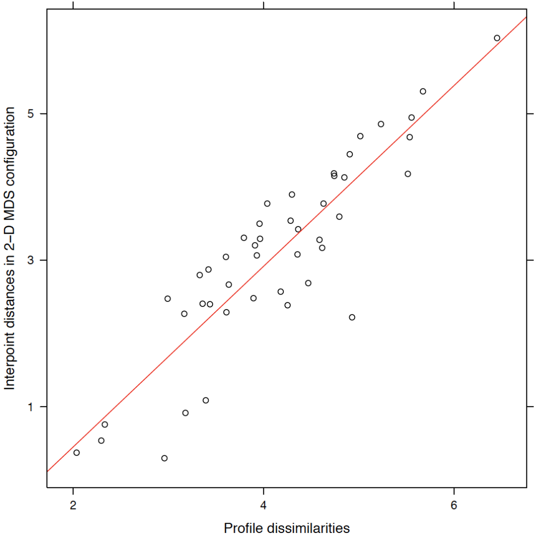

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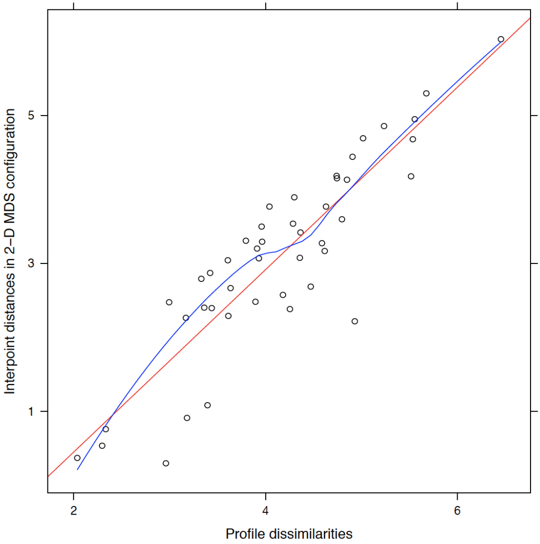

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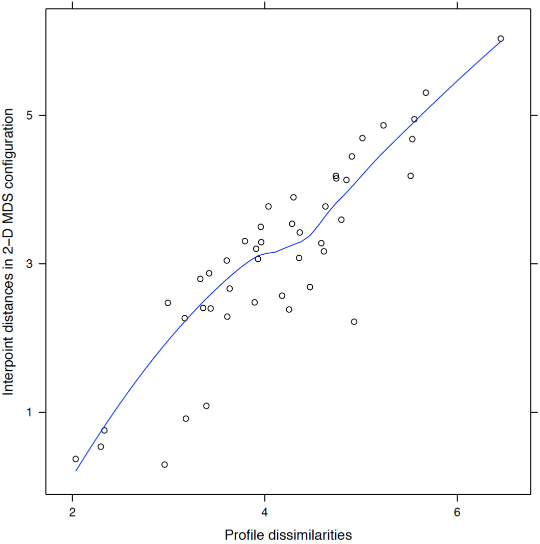


A New Concern

- Distances in MDS solution do not appear to be a linear function of the dissimilarities
- Instead, distances seem to be monotonically related to dissimilarities
- Distances are monotonically related to dissimilarities if, for all i, j, and l, the following holds:

$$\delta_{ij} < \delta_{il} \implies d_{ij} \leq d_{il}$$

Distances Monotonic to Dissimilarities

- Rather than assuming distances should be a linear function of the dissimilarities, perform MDS assuming that d_{ij}'s are a monotonic function of δ_{ij}'s:

$$d_{ij} = f^m(\delta_{ij}) + e_{ij}$$

- In this expression, f^m means "a monotonic function" and e_{ij} is an error term
- But, do we really need to worry about this?
  - What if we simply treated ordinal dissimilarities data as interval-level?
  - To see what happens, we can analyze the rank-order of the driving distances between the ten American cities
  - Again, a useful example because we know the "true" solution

Metric MDS of Ordinal Dissimilarities

Matrix of rank-ordered distances between ten American cities:

Atlanta	1	0	4	22	8	34	6	10	35	36	3
Chicago	2	4	0	13	15	31	21	9	32	30	5
Denver	3	22	13	0	12	11	29	27	16	19	26
Houston	4	8	15	12	0	24	18	25	28	33	23
Los Angeles	5	34	31	11	24	0	39	42	2	17	37
Miami	6	6	21	29	18	39	0	20	44	45	14
New York	7	10	9	27	25	42	20	0	43	40	1
San Francisco	8	35	32	16	28	2	44	43	0	7	41
Seattle	9	36	30	19	33	17	45	40	7	0	38
Washington DC	10	3	5	26	23	37	14	1	41	38	0

Apply Torgerson's double-centering transformation and perform metric MDS

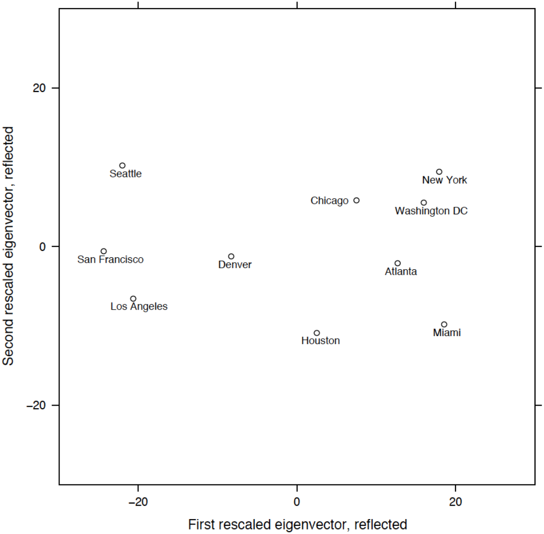
Metric MDS of Ordinal Dissimilarities, cont'd

Double-centered version of rank-ordered distance matrix:

124.6	89.4	-147.6	91.4	-284.6	273.4	212.8	-266.8	-290.0	197.4
89.4	70.2	-17.3	-16.4	-214.3	43.6	195.1	-193.4	-119.2	162.2
-147.6	-17.3	64.2	21.1	202.6	-159.4	-132.0	187.5	147.2	-166.4
91.4	-16.4	21.1	122.0	4.1	128.0	-51.0	-47.6	-187.8	-64.0
-284.6	-214.3	202.6	4.1	462.2	-300.4	-450.4	512.5	382.2	-313.9
273.4	43.6	-159.4	128.0	-300.4	458.0	229.5	-455.6	-487.8	270.6
212.8	195.1	-131.9	-51.0	-450.4	229.5	401.0	-440.6	-303.8	339.5
-266.8	-193.4	187.5	-47.6	512.5	-455.6	-440.6	566.8	554.6	-417.6
-290.0	-119.2	147.2	-187.8	382.2	-487.8	-303.8	554.6	591.4	-286.8
197.4	162.2	-166.4	-64.0	-313.9	270.6	339.5	-417.6	-286.8	279.0

Perform eigendecomposition on double-centered matrix in order to obtain point coordinates.

Metric MDS of Ordinal Dissimilarities, cont'd



Metric MDS of Ordinal Dissimilarities, cont'd

- Goodness of fit for metric MDS solution obtained from ordinal dissimilarities data:

$$Fit = \frac{\sum_{i=1}^{m} \lambda_i^2}{\sum_{i=1}^{q} \lambda_i^2} = \frac{2715.84 + 520.32}{3258.34} = 0.993$$

- A two-dimensional MDS solution "explains" about 99% of the variance in the ordinal dissimilarities
- Point configuration still provides excellent fit, even though there appears to be less information conveyed in the data matrix

Assessment

- Metric MDS of ordinal data seems to work...
- But, it is problematic
  - It is “cheating” with respect to data characteristics
  - It imposes an implicit assumption about relative sizes of differences between dissimilarities
  - Concept of “variance” is undefined for ordinal data
  - Therefore, it is inappropriate to use the eigendecomposition, which maximizes variance explained by successive dimensions
- For these reasons, use a different strategy with ordinal dissimilarities data!

Estimating Metric MDS

- Simple function in "stats" package called "cmdscale()"
  - Carries out Torgerson procedure exactly as we did it "by hand"
  - Very bare bones – returns a list of point coordinate and eigenvalues
  - No way to plot, assess fit in other ways, etc.
- By far the best MDS package is "smacof"
  - Capable of carrying out all the aforementioned variants of MDS
  - Lots of additional functions for converting data to dissimilarities, assessing fit, and producing high quality plots
  - Able to better minimize error even for metric input data by use of a sophisticated algorithm (more later...)